

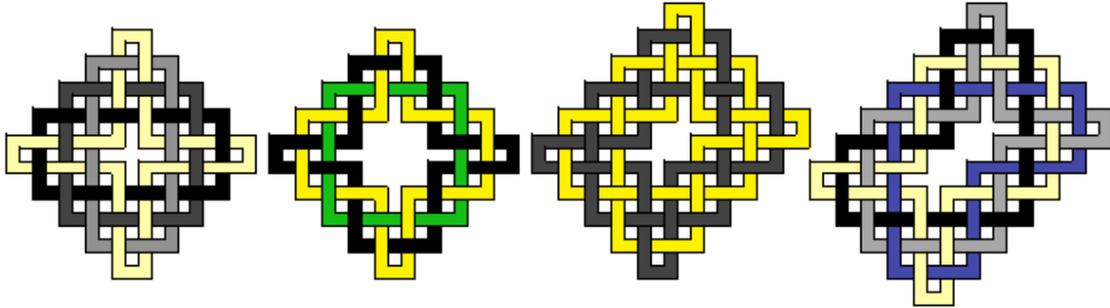
# Celtic Frame Knot/Squared Square Two-Sided Quilt

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**Celtic Frame Knots.** Rectangular borders, or frames of constant width have three parameters: the height  $p$ , the breadth  $q$  and the width (of the band)  $n$ . We refer to such a frame as a  $p \times q$  frame of width  $n$ . Note that  $n$  may be either an integer or a half-integer, such as 1.5.



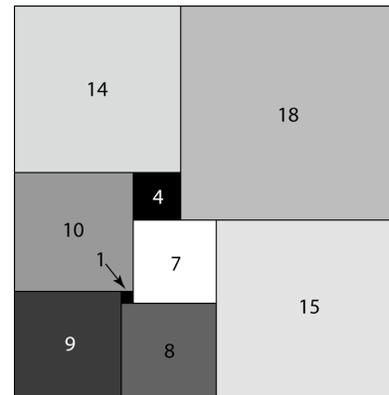
**Examples:** 5 by 5 of width 2, 5 by 5 width of 1.5, 5 by 6 of width 2, and 5 by 7 of width 2

**Theorem:** The number of components in a  $p \times q$  frame of width  $n$  is equal to  $2\gcd(|p - q|, n)$  (if  $n$  is an integer) or to  $\gcd(|p - q|, 2n)$  (if  $n$  is a half-integer  $k + 1/2$ , with  $k$  an integer). Here we require  $2n < \min(p, q)$ .

For proof, see "On the Topology of Celtic Knot Designs" by Gwen Fisher and Blake Mellor, *Proceedings of the 7th Annual Bridges (Mathematical Connections in Art, Music and Science) Conference*, July 2004.

**Squared Rectangles.** A squared rectangle is a rectangle that is tiled by more than one square, where all of the square tiles are different sizes. Consequently, all squared rectangles are not edge-to-edge. The smallest squared square was discovered by A. J. W. Duijvestijn and requires 21 different squares. The smallest squared rectangle was discovered by Z Moron in 1925 and has only 9 squares. This squared rectangle measures 32 by 33 units and has squares with side lengths 1, 4, 7, 8, 9, 10, 14, 15, and 18.

The quilt design uses this smallest squared rectangle as the basis for its layout. Border designs were added to help balance the composition. The design in the top right includes a tiling of a figure commonly used to prove the Pythagorean theorem and is also a common design motif used in traditional Moorish and Amish artwork.



**Figure:** The 32 by 33 squared rectangle used in the quilt

See "Quilt Designs Using Non-Edge-to-Edge Tilings by Squares" by Gwen Fisher, *Proceedings of the Joint Meetings of ISAMA 2003 and the 6th Annual Bridges Conference*, July 2003.